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Fixed points of non-self almost contractions

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ABSTRACT. Let X be a convex metric space, K a non-empty closed subset of X and $T : K \to X$ a non-self almost contraction. Berinde and Păcurar [Berinde, V. and Păcurar, M., *Fixed point theorems for nonself single-valued almost contractions*, Fixed Point Theory, **14** (2013), No. 2, 301–312], proved that if T has the so called property (M) and satisfies Rothe's boundary condition, i.e., maps ∂K (the boundary of K) into K, then T has a fixed point in K. In this paper we observe that property (M) can be removed and, hence, the above fixed point theorem takes place in a different setting.

1. INTRODUCTION

The study of fixed points of single-valued self mappings or multi-valued self mappings satisfying certain contraction conditions has a great majority of results in metric fixed point theory. All these results are mainly generalizations of Banach's contraction principle, see e.g., [1, 17, 20, 21, 22, 32] and references cited therein.

Let (X, d) be a metric space and $T : X \to X$ be a mapping. We say that T is a contraction if there exists $\alpha \in [0, 1)$ such that, for all $x, y \in X$,

$$d(Tx, Ty) \leq \alpha d(x, y).$$

Banach's contraction principle asserts that if T is a contraction and (X, d) is complete, then T has a unique fixed point $x \in X$, and for any $x_0 \in X$ the sequence $\{T^n x_0\}$ converges to x. This result has various non-trivial implications in many branches of pure and applied sciences. Also, it has many applications in solving nonlinear equations, optimization problems and variational inequalities by transforming them in an equivalent fixed point problem.

The fixed point theory for non-self multi-valued mappings developed rapidly after the publication of Assad and Kirk's paper [7] in which they proved a nons-elf multi-valued version of Banach's contraction principle. In 1978, Rhoades [30] obtained the fixed point theorem for non-self single-valued mapping satisfying contractive type condition. Recently, Ćirić et al. [16] proved a fixed point theorem for a class of non-self mappings which satisfy a generalized contraction condition. Further results for non-self mappings were proved in, e.g., [15], [3, 4, 5, 6].

On the other hand, Berinde [8], [10] introduced a new class of self mappings (usually called weak contractions or almost contractions) that satisfy a simple but general contraction condition that includes most of the conditions in Rhoades' classification [29]. He obtained a fixed point theorem for such mappings which generalized the results of Banach, Kannan [19], Chatterjea [14] and Zamfirescu [34].

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Let X be a Banach space, K a nonempty closed subset of X and $T : K \to X$ a non-self mapping. If $x \in K$ is such that $Tx \notin K$, then we can always choose an $y \in \partial K$ (the boundary of K) such that $y = (1-\lambda)x + \lambda Tx \,(0 < \lambda < 1)$, which actually expresses the fact that

$$d(x, Tx) = d(x, y) + d(y, Tx),\ y \in \partial K. \tag{1.1}$$

Definition 1.1. [11]. Let X be a Banach space, K a nonempty closed subset of X and $T : K \to X$ a non-self mapping. Let $x \in K$ with $Tx \notin K$ and let $y \in \partial K$ be the corresponding elements given by (1.1). If, for any such elements x, we have

$$d(y, Ty) \leq d(x, Tx), \tag{1.2}$$

for at least one corresponding $y \in Y$, then we say that T has property (M).

Berinde and Păcurar [11] proved the following result for non-self almost contractions that have property (M).

Theorem 1.1. [11]. *Let X be a Banach space, K a nonempty closed subset of X and $T : K \to X$ a non-self almost contraction. If T has property (M) and satisfies Rothe's boundary condition*

$$T(\partial K) \subset K,$$

then T has a fixed point in K.

In this paper we observe that property (M) can be removed and prove the fixed point theorem in the general setting, thus answering in the affirmative the Open problem in [11].

2. Preliminaries

We recall some basic definitions and preliminaries that will be needed in this paper.

Definition 2.2. A metric space (X, d) is convex if for each $x, y \in X$ with $x \neq y$ there exists $z \in X$, $x \neq z \neq y$, such that

$$d(x, y) = d(x, z) + d(z, y).$$

This definition is similar with the definition of metric space of hyperbolic type. The class of metric spaces of hyperbolic type includes all normed linear spaces and all spaces with hyperbolic metric.

Menger has shown that in a convex metric space each two points are the endpoints of at least one metric segment (see [7]).

Proposition 2.1. [7]. *Let K be a closed subset of a complete and convex metric space X. If $x \in K$ and $y \notin K$, then there exists a point $z \in \partial K$ such that*

$$d(x, y) = d(x, z) + d(z, y).$$

The definition of an almost contraction given by Berinde [9] is as follows.

Definition 2.3. Let (X, d) be a metric space. A map $T : X \to X$ is called almost contraction if there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that

$$d(Tx, Ty) \leq \delta . d(x, y) + L d(y, Tx), \quad \text{for all} \quad x, y \in X. \tag{2.3}$$

Theorem 2.2. [9]. *Let (X,d) be a complete metric space and $T:X\to X$ an almost contraction. Then*

1) $Fix\,(T)=\{x\in X: Tx=x\}\neq\emptyset$;
2) *For any $x_0\in X$, the Picard iteration $\{x_n\}_{n=0}^{\infty}$ converges to some $x^*\in Fix\,(T)$;*
3) *The following estimate holds*

$$d(x_{n+i-1},x^*)\leq\frac{\delta^i}{1-\delta}\,d(x_n,x_{n-1})\,,\quad n=1,2,\ldots;\,i=1,2,\ldots \tag{2.4}$$

Let us recall, see [32], that a mapping T possessing properties 1) and 2) above is called a *weakly Picard operator*.

Notice also that an almost contraction needs not have a unique fixed point, as shown in the following Example (see [11]).

Example 2.1. Let $X=[0,1]$ be the unit interval with the usual norm and let $T:[0,1]\to[0,1]$ be given by $Tx=\dfrac{1}{2}$ for $x\in[0,2/3)$ and $Tx=1$, for $x\in[2/3,1]$. Then T is an almost contraction and it has two fixed points, that is, $FixT=\left\{\dfrac{1}{2},1\right\}$.

3. Main Results

Theorem 3.3. *Let (X,d) be a complete convex metric space and K be a nonempty closed subset of X. Suppose that $T:K\to X$ be a non-self mapping satisfying the following condition*

$$d\,(Tx,Ty)\leq\delta.d\,(x,y)+Ld\,(y,Tx) \tag{3.5}$$

for all $x,y\in K$ and for some $\delta\in(0,1)\,,\,L\geq 0$ such that $(1+L)\,\delta<1$. If T satisfies Rothe's boundary condition, then there exists a fixed point of T in K.
Moreover, if T satisfies the additional condition that there is $\theta\in(0,1)$ and some $L_1\geq 0$ such that

$$d\,(Tx,Ty)\leq\theta.d\,(x,y)+L_1d\,(x,Tx)\,,$$

for all $x,y\in K$, then T has a unique fixed point in K.

Proof. We construct two sequences $\{x_n\}$ and $\{y_n\}$ in the following way. Let $x_0\in K$ and $y_1=Tx_0$. If $y_1\in K$, let $x_1=y_1$. If $y_1\notin K$, then there exists $x_1\in\partial K$ such that

$$d\,(x_0,x_1)+d\,(x_1,y_1)=d\,(x_0,y_1)\,.$$

Thus $x_1\in K$ and we can choose $y_2=Tx_1$. If $y_2\in K$, let $x_2=y_2$. If $y_2\notin K$, then there exists $x_2\in\partial K$ such that

$$d\,(x_1,x_2)+d\,(x_2,y_2)=d\,(x_1,y_2)\,.$$

Continuing the arguments we construct two sequences $\{x_n\}$ and $\{y_n\}$ such that:

(i) $y_{n+1}=Tx_n$;
(ii) $y_n\in K\Longrightarrow y_n=x_n$;
(iii) $y_n\neq x_n$ whenever $y_n\notin K$ and then $x_n\in\partial K$ is such that

$$d\,(x_{n-1},x_n)+d\,(x_n,y_n)=d\,(x_{n-1},y_n)\,.$$

Now we claim that $\{x_n\}$ is a Cauchy sequence. Denote that

$$P=\{x_i\in\{x_n\}:x_i=y_i\}\,,\quad Q=\{x_i\in\{x_n\}:x_i\neq y_i\}\,.$$

Obviously, if $x_n\in Q$, then x_{n-1} and x_{n+1} belong to P. Now, we conclude that there are three possibilities:

Case 1. If $x_n, x_{n+1} \in P$, then $y_n = x_n$, $y_{n+1} = x_{n+1}$. Thus, we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(y_n, y_{n+1}) \\ &= d(Tx_{n-1}, Tx_n) \\ &\leq \delta \cdot d(x_{n-1}, x_n) + Ld(x_n, Tx_{n-1}) \\ &= \delta \cdot d(x_{n-1}, x_n). \end{aligned}$$

Case 2. If $x_n \in P$, $x_{n+1} \in Q$, then $y_n = x_n$, $y_{n+1} \neq x_{n+1}$.We have

$$\begin{aligned} d(x_n, x_{n+1}) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, y_{n+1}) \\ &= d(x_n, y_{n+1}) \\ &= d(y_n, y_{n+1}) \\ &= d(Tx_{n-1}, Tx_n) \\ &\leq \delta \cdot d(x_{n-1}, x_n) + Ld(x_n, Tx_{n-1}) \\ &\leq \delta \cdot d(x_{n-1}, x_n). \end{aligned}$$

Case 3. $x_n \in Q$, $x_{n+1} \in P$, then $x_{n-1} \in P$, $y_n \neq x_n$, $y_{n+1} = x_{n+1}$, $y_{n-1} = x_{n-1}$ and $y_n = Tx_{n-1}$. We have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(x_n, y_{n+1}) \\ &\leq d(x_n, y_n) + d(y_n, y_{n+1}) \\ &= d(x_n, y_n) + d(Tx_{n-1}, Tx_n) \\ &\leq d(x_n, y_n) + \delta . d(x_{n-1}, x_n) + Ld(x_n, Tx_{n-1}) \\ &< d(x_n, y_n) + d(x_{n-1}, x_n) + Ld(x_n, y_n) \\ &= d(x_{n-1}, y_n) + Ld(x_n, y_n) \\ &= d(x_{n-1}, y_n) + Ld(x_{n-1}, y_n) - Ld(x_{n-1}, x_n) \\ &< d(x_{n-1}, y_n) + Ld(x_{n-1}, y_n) \end{aligned}$$

since $\delta < 1$, $L > 0$. Therefore

$$\begin{aligned} d(x_n, x_{n+1}) &< (1 + L) d(x_{n-1}, y_n) \\ &= (1 + L) d(y_{n-1}, y_n) \\ &= (1 + L) d(Tx_{n-2}, Tx_{n-1}) \\ &< (1 + L) \delta d(x_{n-2}, x_{n-1}) + (1 + L) Ld(x_{n-1}, Tx_{n-2}) \\ &= (1 + L) \delta d(x_{n-2}, x_{n-1}). \end{aligned}$$

Since

$$h = (1 + L) \delta < 1,$$

then

$$d(x_n, x_{n+1}) < hd(x_{n-2}, x_{n-1})$$

Thus, combining Cases 1, 2, and 3, it follows that

$$d(x_n, x_{n+1}) \leq \alpha \omega_n,$$

where

$$\begin{aligned} \alpha &= \max\{\delta, h\} \\ &= h, \end{aligned}$$

and

$$\omega_n = \max\{d(x_{n-1}, x_n), d(x_{n-2}, x_{n-1})\}.$$

Following [7], by induction it follows that for $n > 1$

$$d(x_n, x_{n+1}) \leq h^{(n-1)/2}\omega_2,$$

where

$$\omega_2 = \max\{d(x_2, x_1), d(x_0, x_1)\}.$$

Now, for $n > m$, we have

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + ... + d(x_{m+1}, x_m) \\ &\leq \left(h^{(n-1)/2} + h^{(n-2)/2} + ... + h^{(m-1)/2}\right)\omega_2. \end{aligned}$$

This implies that the sequence $\{x_n\}$ is a Cauchy sequence. Since X is complete and K is closed, it follows that there exists $z \in K$ such that

$$z = \lim_{n\to\infty} x_n.$$

By construction of $\{x_n\}$, there is a subsequence $\{x_q\}$ such that

$$y_q = x_q = Tx_{q-1}.$$

We shall prove that $Tz = z$. In fact,

$$d(Tx_{q-1}, Tz) \leq \delta d(x_{q-1}, z) + Ld(z, Tx_{q-1})$$

Taking the limit as $q \to \infty$, we get $d(z, Tz) = 0$ and hence $Tz = z$, which shows that z is the fixed point of T.

To prove that z is a unique fixed point of T, suppose that z, w are two fixed points of T. Then, we have

$$\begin{aligned} d(z, w) &= d(Tz, Tw) \\ &\leq \theta d(z, w) + L_1 d(z, Tz) \\ &= \theta d(z, w), \end{aligned}$$

which is a contradiction since $\theta < 1$. Thus $d(Tz, z) = 0$, and hence z is a unique fixed point of T. □

Remark 3.1. Note that the mapping T in the previous theorem is not supposed to have property (M). Instead, we assume a certain relationship between the constants appearing in the almost contraction condition, which is not very restrictive, as shown by the next example.

Example 3.2. Let X be the set of real numbers with the usual norm, $K = [0, 1]$ be the unit interval and let $T : [0, 1] \to \mathbb{R}$ be given by $Tx = \frac{1}{9}x$ for $x \in [0, 1/2)$, $T\left(\frac{1}{2}\right) = -1$, and $Tx = \frac{1}{9}x + \frac{8}{9}$, for $x \in (1/2, 1]$.

Then T has two fixed points, that is, $Fix(T) = \{0, 1\}$ and T is an almost contraction. Indeed, we have 8 possible cases:

Case 1: $x \in [0, 1/2), y \in (1/2, 1]$. Then condition (3.5) reduces to

$$\left|\frac{1}{9}x - \frac{1}{9}y - \frac{8}{9}\right| \leq \delta |x - y| + L\left|y - \frac{1}{9}x\right|.$$

Since $\left|\frac{1}{9}x - \frac{1}{9}y - \frac{8}{9}\right| \leq 1$ and $\left|y - \frac{1}{9}x\right| \geq \frac{4}{9}$, in order to have the previous inequality satisfied, we need to take $L \geq \frac{9}{4}$ and $0 < \delta < 1$.

Case 2: $y \in [0, 1/2), x \in (1/2, 1]$. Then condition (3.5) reduces to

$$\left|\frac{1}{9}x + \frac{8}{9} - \frac{1}{9}y\right| \leq \delta\,|x - y| + L\left|y - \frac{1}{9}x - \frac{8}{9}\right|.$$

Since $\left|\frac{1}{9}x + \frac{8}{9} - \frac{1}{9}y\right| \leq 1$ and $\left|y - \frac{1}{9}x - \frac{8}{9}\right| \geq \frac{4}{9}$, in order to have the previous inequality satisfied, we need to take $L \geq \frac{9}{4}$ and $0 < \delta < 1$.

Case 3: $x, y \in [0, 1/2)$. Then condition (3.5) reduces to

$$\left|\frac{1}{9}x - \frac{1}{9}y\right| \leq \delta\,|x - y| + L\left|y - \frac{1}{9}x\right|.$$

Since $\left|\frac{1}{9}x - \frac{1}{9}y\right| \leq \frac{1}{18}$ and $\left|y - \frac{1}{9}x\right| \geq 0$, in order to have the previous inequality satisfied, we need to take $L \geq 0$ and $0 < \delta < \frac{1}{9}$.

Case 4: $x, y \in (1/2, 1]$. Then condition (3.5) reduces to

$$\left|\frac{1}{9}x + \frac{8}{9} - \frac{1}{9}y - \frac{8}{9}\right| \leq \delta\,|x - y| + L\left|y - \frac{1}{9}x - \frac{8}{9}\right|.$$

Since $\left|\frac{1}{9}x - \frac{1}{9}y\right| \leq \frac{1}{18}$ and $\left|y - \frac{1}{9}x - \frac{8}{9}\right| \geq 0$, in order to have the previous inequality satisfied, we need to take $L \geq 0$ and $0 < \delta < \frac{1}{9}$.

Case 5: $x = \frac{1}{2}, y \in [0, 1/2)$. Then condition (3.5) reduces to

$$\left|-1 - \frac{1}{9}y\right| \leq \delta\left|\frac{1}{2} - y\right| + L\,|y + 1\,|.$$

Since $\left|-1 - \frac{1}{9}y\right| \leq \frac{19}{18}$ and $|y + 1\,| \geq 1$, in order to have the previous inequality satisfied, we need to take $L \geq \frac{19}{18}$ and $0 < \delta < 1$.

Case 6: $x \in [0, 1/2), y = \frac{1}{2}$. Then condition (3.5) reduces to

$$\left|\frac{1}{9}x + 1\right| \leq \delta\left|x - \frac{1}{2}\right| + L\left|\frac{1}{2} - \frac{1}{9}x\right|.$$

Since $\left|\frac{1}{9}x + 1\right| \leq \frac{19}{18}$ and $\left|\frac{1}{2} - \frac{1}{9}x\right| \geq \frac{4}{9}$, in order to have the previous inequality satisfied, we need to take $L \geq \frac{19}{8}$ and $0 < \delta < 1$.

Case 7: $x = \frac{1}{2}, y \in (1/2, 1]$. Then condition (3.5) reduces to

$$\left|-1 - \frac{1}{9}y - \frac{8}{9}\right| \leq \delta\left|\frac{1}{2} - y\right| + L\,|y + 1\,|.$$

Since $\left|-1-\frac{1}{9}y-\frac{8}{9}\right| \le 2$ and $|y+1| \ge \frac{3}{2}$, in order to have the previous inequality satisfied, we need to take $L \ge \frac{4}{3}$ and $0<\delta<1$.

Case 8: $x \in (1/2, 1], y = \frac{1}{2}$. Then condition (3.5) reduces to

$$\left|\frac{1}{9}x+\frac{8}{9}+1\right| \le \delta\left|x-\frac{1}{2}\right| + L\left|\frac{1}{2}-\frac{1}{9}x-\frac{8}{9}\right|.$$

Since $\left|\frac{1}{9}x+\frac{8}{9}+1\right| \le 2$ and $\left|\frac{1}{2}-\frac{1}{9}x-\frac{8}{9}\right| \ge \frac{4}{9}$, in order to have the previous inequality satisfied, we need to take $L \ge \frac{9}{2}$ and $0<\delta<1$.

By summarizing all possible cases, we conclude that T is an almost contraction with $L=\frac{9}{2}$, and $\delta=\frac{1}{10}$, which satisfy the condition in our theorem: $\delta(1+L)<1$.

By Theorem 8 we obtain as a particular case, a fixed point theorem for non-self Banach contractions. For other related results and various possible further developments and applications, the reader is referred to [2], [12], [13], [18], [23]-[27], [31].

Corollary 3.1. *Let (X,d) be a complete convex metric space and K be a nonempty closed subset of X. Suppose that $T:K\to X$ be a non-self mapping satisfying the following condition*

$$d(Tx,Ty) \le \delta \cdot d(x,y),$$

for all $x,y \in K$ where $\delta \in (0,1)$. If T satisfies Rothe's boundary condition, then there exists a unique fixed point of T in K.

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