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Dedicated to Prof. Billy E. Rhoades on the occasion of his 90^{th} anniversary

Strong convergence of Picard and Mann iterations for strongly demicontractive multi-valued mappings

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ABSTRACT. A class of demicontractive mappings was first introduced in [Hicks, T. L. and Kubicek, J. D., *On the Mann iteration process in a Hilbert space*, J. Math. Anal. Appl., **59** (1977) 498–504 and Măruşter, Ş., *The solution by iteration of nonlinear equations in Hilbert spaces*, Proc. Amer. Math. Soc., **63** (1977), 69–73] and was first mentioned in the case of multi-valued mappings in [Chidume, C. E., Bello, A. U. and Ndambomve, P., *Strong and Δ-convergence theorems for common fixed points of a finite family of multivalued demicontractive mappings in CAT(0) spaces*, Abstr. Appl. Anal., **2014** (2014), https://doi.org/10.1155/2014/805168 and Isiogugu, F. O. and Osilike, M. O., *Convergence theorems for new classes of multivalued hemicontractive-type mappings*, Fixed Point Theory Appl., **2014** (2014), https://doi.org/10.1186/1687-1812-2014-93]. The demicontractivity with some weak smoothness conditions ensures only weak convergence of Mann iteration. In 2015, Măruşter and Rus [*Kannan contractions and strongly demicontractive mappings*, Creat. Math. Inform., **24** (2015), No. 2, 173–182], introduced a class of strongly demicontractive mappings, and also discussed some relationships between strongly demicontractive mappings and Kannan contractions. In this paper, we introduce a new class of strongly demicontractive multi-valued mappings in Hilbert spaces. Strong convergence theorems of Picard and Mann iterative methods for strongly demicontractive multi-valued mappings are established under some suitable coefficients and control sequences.

1. INTRODUCTION AND PRELIMINARIES

Throughout this paper, we assume that $\mathcal{H}$ is a real Hilbert space with an inner product $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|$.

Let C be a nonempty closed convex subset of $\mathcal{H}$. A fixed point of a mapping $T : C \to C$ is a point in C, which is mapped to itself by T, and the set of all fixed points of T is denoted by $Fix(T) := \{x \in C : x = Tx\}$. Picard and Mann iterative methods are well-known procedures, in order to approximate fixed points of the mapping T. Let us recall these procedures:

$$\text{(1.1)} \qquad \textit{Picard Iteration} \qquad \begin{cases} x_1 \in C, \\ x_{n+1} = Tx_n,\ n \geq 1, \end{cases}$$

$$\text{(1.2)} \qquad \textit{Mann Iteration} \qquad \begin{cases} x_1 \in C, \\ x_{n+1} = (1-t_n)x_n + t_n Tx_n,\ n \geq 1, \end{cases}$$

where $\{t_n\}$ is a sequence of real numbers. In particular, if $t_n \equiv t$ is constant, the iteration (1.2) reduces to the so-called Krasnoselskij iteration. It is well known that Picard iteration converges faster than Mann iteration for some classes of nonlinear mappings; however,

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the Mann iteration has been mainly introduced, in order to approximate fixed points of those mappings, for which the Picard iteration does not converge.

In 1973–1977, the concept of the demicontractivity of nonlinear mappings was introduced in [6, 12, 13] as follows: A mapping $T : C \to C$ is called *demicontractive* if $Fix(T) \neq \emptyset$ and there exists $\kappa \in [0, 1)$ such that

$$\|Tx - p\|^2 \leq \|x - p\|^2 + \kappa\|x - Tx\|^2,\ \forall x \in C,\ p \in Fix(T). \tag{1.3}$$

A class of demicontractive mappings has been widely studied (see, [1, 4, 7, 9, 14, 19, 20]) due to this class includes several common types of classes of mappings occurring in nonlinear analysis and fixed point problems, such as a class of quasi-nonexpansive mappings. However, the demicontractivity with some weak smoothness conditions guarantees only weak convergence of Mann iteration, see [4, 6, 13]. For this reason, the problem of additional conditions for strong convergence of Mann iteration was discussed by many authors (see, for instance, [1, 4, 6, 13, 15]).

In 2015, Măruşter and Rus [16] introduced a class of *strongly demicontractive* mappings $T : C \to C$ defined by the conditions: $Fix(T) \neq \emptyset$ and

$$\|Tx - p\|^2 \leq \alpha\|x - p\|^2 + \kappa\|x - Tx\|^2,\ \forall x \in C,\ p \in Fix(T), \tag{1.4}$$

where $\alpha \in (0, 1)$ and $\kappa \in [0, 1)$. Obviously, the class of strongly demicontractive mappings is contained in the calss of demicontractive mappings. In [16], some relationships between strongly demicontractive mappings and Kannan contractions were discussed, and then strong convergence results of Picard and Krasnoselskij iterations were obtained by Kannan's fixed point theorem [11]. Various classes of mappings defined similar to the class of strongly demicontractive mappings, were studied by many authors, see [2, 5, 17, 18].

Next, we will review some definitions and notations on multi-valued mappings. Let C be a nonempty closed convex subset of $\mathcal{H}$, and $T : C \to 2^C$ a multi-valued mapping. An element $p \in C$ is called a fixed point of T if $p \in Tp$. The set of all fixed points of T is also denoted by $Fix(T)$. We say that T satisfies the endpoint condition if $Tp = \{p\}$ for all $p \in Fix(T)$. The Pompeiu-Hausdorff metric on $CB(C)$ is defined by

$$H(A, B) := \max\left\{\sup_{x\in A} D(x, B), \sup_{y\in B} D(y, A)\right\}$$

for all $A, B \in CB(C)$, where $CB(C)$ denotes the family of all nonempty closed bounded subsets of C and $D(x, B) := \inf\{\|x - y\| : y \in B\}$.

In [3, 8], they first introduced a class of *demicontractive multi-valued mappings* in the following way: A multi-valued mapping $T : C \to CB(C)$ is said to be *demicontractive* if $Fix(T) \neq \emptyset$, and there exists $\kappa \in [0, 1)$ such that

$$H(Tx, Tp)^2 \leq \|x - p\|^2 + \kappa D(x, Tx)^2,\ \forall x \in C,\ p \in Fix(T) \tag{1.5}$$

(in particular, if $\kappa = 0$, then T is called *quasi-nonexpansive*). Also, weak convergence theorems of the Mann-type iteration for demicontractive multi-valued mappings were proved under the demiclosedness principle and the endpoint condition, see [3, 8].

We now introduce a new class of *strongly demicontractive multi-valued mappings* in the terminology of Măruşter and Rus [16].

Definition 1.1. A multi-valued mapping $T : C \to CB(C)$ is said to be *strongly demicontractive* if $Fix(T) \neq \emptyset$, and there exist $\alpha \in (0, 1)$ and $\kappa \in [0, 1)$ such that

$$H(Tx, Tp)^2 \leq \alpha\|x - p\|^2 + \kappa D(x, Tx)^2,\ \forall x \in C,\ p \in Fix(T). \tag{1.6}$$

If T is a strongly demicontractive multi-valued mapping having a fixed point p such that $Tp = \{p\}$, then T has a unique fixed point p. In fact, if $q \in Fix(T)$, then

$$\|q-p\|^2 = D(q,Tp)^2 \leq H(Tq,Tp)^2 \leq \alpha\|q-p\|^2 + \kappa D(q,Tq)^2 = \alpha\|q-p\|^2,$$

which implies $q = p$.

The following is an example of strongly demicontractive multi-valued mappings.

Example 1.1. ([10]) Let $\mathcal{H} = \mathbb{R}$. For each $i \geq 0$, define $T_i : \mathbb{R} \to 2^{\mathbb{R}}$ by

$$T_i x = \begin{cases} \big[-\frac{2i+1}{2}x, -(i+1)x\big] & \text{if } x \leq 0, \\ \big[-(i+1)x, -\frac{2i+1}{2}x\big] & \text{if } x > 0. \end{cases}$$

We see that $Fix(T_i) = \{0\}$. Let $\alpha \in (0,1)$. Thus,

$$H(T_i x, T_i 0)^2 = |-(i+1)x - 0|^2 = (i+1)^2|x-0|^2 = \alpha|x-0|^2 + \big((i+1)^2 - \alpha\big)|x|^2$$

and

$$D(x,T_i x)^2 = \left|x - \left(-\frac{2i+1}{2}x\right)\right|^2 = \frac{(2i+3)^2}{4}|x|^2.$$

It follows that

$$H(T_i x, T_i 0)^2 = \alpha|x-0|^2 + \frac{4\left((i+1)^2-\alpha\right)}{(2i+3)^2}D(x,T_i x)^2.$$

Hence, T_i is strongly demicontractive for any $\alpha \in (0,1)$ and $\kappa_i = \frac{4\left((i+1)^2-\alpha\right)}{(2i+3)^2} \in (0,1)$.

In Example 1.1, we note that T_i is not quasi-nonexpansive for all $i \geq 1$.

In this paper, inspired and motivated by the results of Măruşter and Rus [16] and above-mentioned works, we are interested to study some sufficient conditions for strong convergence of Picard-type and Mann-type iterations for strongly demicontractive multi-valued mappings in Hilbert spaces. We establish strong convergence theorems of the Picard and Mann iterative methods without any strong conditions, but with some restrictions on the coefficients α, κ and the control sequences.

2. Strong convergence theorems

In this section, we prove strong convergence results of Picard and Mann iterative methods for strongly demicontractive multi-valued mappings.

2.1. **Picard Iteration.**

Theorem 2.1. *Let $\mathcal{H}$ be a real Hilbert space and C a nonempty closed convex subset of $\mathcal{H}$. Let $T : C \to CB(C)$ be a strongly demicontractive mapping with coefficients α, κ such that $0 < \alpha < 1/9$ and $0 \leq \kappa < (1 + 9\alpha - 6\sqrt{\alpha})/4$. Suppose that $\{x_n\}$ is a sequence generated by the following Picard iteration:*

$$\begin{cases} x_1 \in C, \\ x_{n+1} \in Tx_n, \ n \geq 1. \end{cases} \tag{2.7}$$

If T has a fixed point x^* such that $Tx^* = \{x^*\}$, then the sequence $\{x_n\}$ converges strongly to x^*.

Proof. By the strongly demicontractivity of T, we have

$$\begin{aligned}\|x_{n+1}-x^*\|^2 &= D(x_{n+1},Tx^*)^2\\ &\leq H(Tx_n,Tx^*)^2\\ &\leq \alpha\|x_n-x^*\|^2+\kappa D(x_n,Tx_n)^2\\ &\leq \alpha\|x_n-x^*\|^2+\kappa\|x_n-x_{n+1}\|^2\\ &\leq \left(\sqrt{\alpha}\|x_n-x^*\|+\sqrt{\kappa}\|x_n-x_{n+1}\|\right)^2,\end{aligned}$$

it follows that

$$\begin{aligned}\|x_{n+1}-x^*\| &\leq \sqrt{\alpha}\|x_n-x^*\|+\sqrt{\kappa}\|x_n-x_{n+1}\|\\ &\leq \sqrt{\alpha}\left(\|x_n-x_{n+1}\|+\|x_{n+1}-x^*\|\right)+\sqrt{\kappa}\|x_n-x_{n+1}\|\\ &= \left(\sqrt{\alpha}+\sqrt{\kappa}\right)\|x_n-x_{n+1}\|+\sqrt{\alpha}\|x_{n+1}-x^*\|,\end{aligned}$$

which implies that

$$\|x_{n+1}-x^*\| \leq \frac{\sqrt{\alpha}+\sqrt{\kappa}}{1-\sqrt{\alpha}}\|x_n-x_{n+1}\|. \tag{2.8}$$

Take $\delta := \frac{\sqrt{\alpha}+\sqrt{\kappa}}{1-\sqrt{\alpha}}$. From (2.8), we get

$$\begin{aligned}\|x_n-x_{n+1}\| &\leq \|x_n-x^*\|+\|x_{n+1}-x^*\|\\ &\leq \delta\left(\|x_{n-1}-x_n\|+\|x_n-x_{n+1}\|\right),\end{aligned}$$

which implies that

$$\|x_n-x_{n+1}\| \leq \frac{\delta}{1-\delta}\|x_{n-1}-x_n\|. \tag{2.9}$$

By (2.9), we have

$$\begin{aligned}\|x_n-x_{n+1}\| &\leq \left(\frac{\delta}{1-\delta}\right)^2\|x_{n-2}-x_{n-1}\|\\ &\leq \left(\frac{\delta}{1-\delta}\right)^3\|x_{n-3}-x_{n-2}\|\\ &\vdots\\ &\leq \left(\frac{\delta}{1-\delta}\right)^{n-1}\|x_1-x_2\|.\end{aligned} \tag{2.10}$$

If $m \geq n$, then it follows from (2.10) that

$$\begin{aligned}\|x_n-x_m\| &\leq \|x_n-x_{n+1}\|+\|x_{n+1}-x_{n+2}\|+\ldots+\|x_{m-1}-x_m\|\\ &\leq \left(\frac{\delta}{1-\delta}\right)^{n-1}\|x_1-x_2\|+\left(\frac{\delta}{1-\delta}\right)^{n}\|x_1-x_2\|+\ldots+\left(\frac{\delta}{1-\delta}\right)^{m-2}\|x_1-x_2\|\\ &= \left(\frac{\delta}{1-\delta}\right)^{n-1}\left\{1+\left(\frac{\delta}{1-\delta}\right)+\left(\frac{\delta}{1-\delta}\right)^2+\ldots+\left(\frac{\delta}{1-\delta}\right)^{m-n-1}\right\}\|x_1-x_2\|\\ &\leq \left(\frac{\delta}{1-\delta}\right)^{n-1}\left(\frac{1-\delta}{1-2\delta}\right)\|x_1-x_2\|.\end{aligned} \tag{2.11}$$

By the conditions of α and κ, we get $0 < \delta = \frac{\sqrt{\alpha}+\sqrt{\kappa}}{1-\sqrt{\alpha}} < 1/2$, and hence $0 < \frac{\delta}{1-\delta} < 1$. Thus, $\left(\frac{\delta}{1-\delta}\right)^{n-1}\left(\frac{1-\delta}{1-2\delta}\right)\|x_1 - x_2\| \to 0$ as $n \to \infty$, and it follows from (2.11) that $\{x_n\}$ is a Cauchy sequence in $\mathcal{H}$. By the completeness of a Hilbert space $\mathcal{H}$, then there exists a point $p \in \mathcal{H}$ such that $x_n \to p$ as $n \to \infty$. Since $\{x_n\} \subset C$ and C is closed, we have $p \in C$. From (2.8), we have

$$\|p - x^*\| \leq \|p - x_n\| + \|x_n - x^*\| \leq \|p - x_n\| + \delta\|x_{n-1} - x_n\| \to 0$$

as $n \to \infty$. Therefore, $p = x^*$. This completes the proof. □

2.2. **Mann Iteration.**

Theorem 2.2. *Let $\mathcal{H}$ be a real Hilbert space and C a nonempty closed convex subset of $\mathcal{H}$. Let $T : C \to CB(C)$ be a strongly demicontractive mapping with coefficients α, κ such that $\frac{1}{1-\alpha} < \frac{4(1-\kappa)}{3}$. Suppose that $\{x_n\}$ is a sequence generated by the following Mann iteration:*

$$\begin{cases} x_1 \in C, \\ x_{n+1} = (1-t_n)x_n + t_n y_n, \ n \geq 1, \end{cases} \tag{2.12}$$

where $y_n \in Tx_n$, and $\frac{1}{1-\alpha} \leq t_n \leq \varepsilon < \frac{4(1-\kappa)}{3}$. If T has a fixed point x^* such that $Tx^* = \{x^*\}$, then the sequence $\{x_n\}$ converges strongly to x^*.

Proof. Since T is strongly demicontractive, we have

$$\begin{aligned}
\langle x_n - y_n, x_n - x^*\rangle &= \frac{1}{2}\left(\|x_n - y_n\|^2 + \|x_n - x^*\|^2 - \|y_n - x^*\|^2\right) \\
&= \frac{1}{2}\left(\|x_n - y_n\|^2 + \|x_n - x^*\|^2 - D(y_n, Tx^*)^2\right) \\
&\geq \frac{1}{2}\left(\|x_n - y_n\|^2 + \|x_n - x^*\|^2 - H(Tx_n, Tx^*)^2\right) \\
&\geq \frac{1}{2}\left(\|x_n - y_n\|^2 + \|x_n - x^*\|^2 - \alpha\|x_n - x^*\|^2 - \kappa D(x_n, Tx_n)^2\right) \\
&\geq \frac{1-\alpha}{2}\|x_n - x^*\|^2 + \frac{1-\kappa}{2}\|x_n - y_n\|^2.
\end{aligned} \tag{2.13}$$

From (2.13), we have

$$\begin{aligned}
\|x_{n+1} - x^*\|^2 &= \|(x_n - x^*) - t_n(x_n - y_n)\|^2 \\
&= \|x_n - x^*\|^2 - 2t_n\langle x_n - y_n, x_n - x^*\rangle + t_n^2\|x_n - y_n\|^2 \\
&\leq \|x_n - x^*\|^2 - t_n(1-\alpha)\|x_n - x^*\|^2 - t_n(1-\kappa)\|x_n - y_n\|^2 \\
&\quad + t_n^2\|x_n - y_n\|^2 \\
&= (1 - t_n + t_n\alpha)\|x_n - x^*\|^2 + \frac{t_n^2 - t_n + t_n\kappa}{t_n^2}\|x_n - x_{n+1}\|^2.
\end{aligned}$$

Since $1 - t_n + t_n\alpha \leq 0$, then above inequality yields

$$\|x_{n+1} - x^*\| \leq \frac{\sqrt{t_n^2 - t_n + t_n\kappa}}{t_n}\|x_n - x_{n+1}\|. \tag{2.14}$$

Take $\delta := \frac{\sqrt{\varepsilon^2-\varepsilon+\varepsilon\kappa}}{\varepsilon}$. From (2.14), we get

$$\begin{aligned}\|x_n - x_{n+1}\| &\le \|x_n - x^*\| + \|x_{n+1} - x^*\| \\ &\le \frac{\sqrt{t_{n-1}^2 - t_{n-1} + t_{n-1}\kappa}}{t_{n-1}}\|x_{n-1} - x_n\| + \frac{\sqrt{t_n^2 - t_n + t_n\kappa}}{t_n}\|x_n - x_{n+1}\| \\ &\le \delta(\|x_{n-1} - x_n\| + \|x_n - x_{n+1}\|).\end{aligned}$$

Since $1 < \varepsilon < \frac{4(1-\kappa)}{3}$, we get $0 < \delta = \frac{\sqrt{\varepsilon^2-\varepsilon+\varepsilon\kappa}}{\varepsilon} < 1/2$. By the same proof as (2.9)–(2.11) in Theorem 2.1, we deduce that $\{x_n\}$ converges to a point $p \in C$. Then, by (2.14),

$$\|p - x^*\| \le \|p - x_n\| + \|x_n - x^*\| \le \|p - x_n\| + \delta\|x_{n-1} - x_n\| \ \to 0$$

as $n \to \infty$. Hence $x_n \to x^* \in Fix(T)$ as $n \to \infty$. □

Remark 2.1. The conditions of the coefficients α, κ of a strongly demicontractive mapping in Theorem 2.2 can be formulated in the following equivalent form:

$$0 < \alpha < \frac{1}{4} \text{ and } 0 \le \kappa < \frac{1-4\alpha}{4(1-\alpha)}, \tag{2.15}$$

and these conditions are somewhat weaker than the conditions of α and κ in Theorem 2.1.

3. A Numerical Example

In this section, we give a numerical result for supporting our main results.

Example 3.2. Let $\mathcal{H} = \mathbb{R}$ and $C = [-0.24, 0.24]$. Let $T : C \to CB(C)$ be defined by

$$Tx = \begin{cases} \big[-3.5x^3 + 0.1035, -7x^3 + 0.107\big] & \text{if } x < 0.1, \\ \big[-7x^3 + 0.107, -3.5x^3 + 0.1035\big] & \text{if } x \ge 0.1. \end{cases}$$

The mapping T is strongly demicontractive with a unique fixed point $x^* = 0.1$ and constants $\alpha = 0.00004$, $\kappa = 0.24$. We consider the Picard iteration (2.7) by choosing

$$x_{n+1} = \begin{cases} -7x_n^3 + 0.107 & \text{if } x_n < 0.1, \\ -3.5x_n^3 + 0.1035 & \text{if } x_n \ge 0.1, \end{cases}$$

and the Mann iteration (2.12) by choosing

$$y_n = \begin{cases} -7x_n^3 + 0.107 & \text{if } x_n < 0.1, \\ -3.5x_n^3 + 0.1035 & \text{if } x_n \ge 0.1, \end{cases} \text{ and } t_n = 1.005.$$

Now, we choose the initial point by $x_1 = -0.2$, and then the numerical experiment of the iterative methods (2.7) and (2.12) is shown by Table 1.

Remark 3.2. By testing the convergence behavior of the Picard iteration (2.7) and the Mann iteration (2.12) in Example 3.2, we note that

(i) Both methods converge to $0.1 \in Fix(T)$;
(ii) The Picard iteration (2.7) converges faster than the Mann iteration (2.12).

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	Picard Iteration (2.7)		Mann Iteration (2.12)	
n	x_n	$\lvert x_n - x^*\rvert$	x_n	$\lvert x_n - x^*\rvert$
1	-0.2	0.3	-0.2	0.3
2	0.163	0.063	0.164815	0.064815
3	0.0883424	0.0116576	0.0874455	0.0125545
4	0.1021738	0.0021738	0.1023937	0.0023937
5	0.0997668	0.0002332	0.0997293	0.0002707
6	0.1000489	0.0000489	0.1000583	0.0000583
7	0.0999949	0.0000051	0.0999936	0.0000064
8	0.1000011	0.0000011	0.1000014	0.0000014
9	0.0999999	0.0000001	0.0999998	0.0000002
10	0.10000002	2.377E-08	0.10000003	3.327E-08

TABLE 1. Numerical experiment of the procedures (2.7) and (2.12)

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