Journal of Computational and Applied Mathematics 311 (2017) 718–720

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Corrigendum

Corrigendum to: "Coincidence and fixed points for multi-valued mappings and its application to nonconvex integral inclusions" [J. Comput. Appl. Math. 283 (2015) 201–217]

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ARTICLE INFO

Article history:
Received 3 June 2016

ABSTRACT

In a recent paper [H.K. Pathak, R.P. Agarwal, Y.J. Cho, Coincidence and fixed points for multi-valued mappings and its application to nonconvex integral inclusions, J. Comput. Appl. Math. 283 (2015) 201–217.], the authors have studied some problems on coincidence points and fixed points of multi-valued mappings.

In order to illustrate the generality of their main result (Theorem 3.2) with respect to older related results (Nadler's, Berinde–Berinde, Mizoguchi–Takahashi and Du's fixed point theorem), the authors presented two examples, i.e., Examples 3.3 and 3.4.

The main aim of this note is to show that Example 3.3 fails in proving the generality of Theorem 3.2 over Berinde–Berinde's fixed point theorem.

1. Introduction

In [1], Pathak, Agarwal and Cho have studied some problems concerning the existence of coincidence points and fixed points of multi-valued mappings.

By using some characterizations of $\mathcal{P}$-functions, they established some new existence theorems for coincidence points and fixed points of multi-valued mappings, results that are distinct from Nadler's fixed point theorem [2], Berinde–Berinde's fixed point theorem [3], Mizoguchi–Takahashi's fixed point theorem [4] and Du's fixed point theorem [5]. The new results were then applied to some nonconvex integral inclusion problems.

To illustrate the generality of Theorem 3.2 over Berinde–Berinde's fixed point theorem as well as over Mizoguchi–Takahashi's fixed point theorem and Du's fixed point theorem, the authors give some appropriate examples (Examples 3.3 and 3.4).

DOI of original article: http://dx.doi.org/10.1016/j.cam.2014.12.019.

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http://dx.doi.org/10.1016/j.cam.2016.08.006

The main aim of this note is to show that Example 3.3 fails in proving the generality of Theorem 3.2 in [1] over Berinde–Berinde's fixed point theorem [3]. So, the problem whether Theorem 3.2 is more general than Berinde–Berinde's fixed point theorem remains open. Another open problem related to this topic is also stated.

To make this note as much as possible self-contained, we present in the next section some concepts and results needed for our exposition.

2. Fixed point theorems for multi-valued mappings

Let (X, d) be a metric space. We denote by $\mathcal{N}(X)$ the class of all nonempty subsets of X, by $\mathcal{CB}(X)$ the class of all nonempty closed and bounded subsets of X and by $\mathcal{K}(X)$ the class of all nonempty compact subsets of X. For $A, B \subset X$, we consider

$$D(A, B) = \inf\{d(a, b) : a \in A, b \in B\},$$
$$D(a, B) = D(\{a\}, B) = \inf\{d(a, b) : b \in B\},$$

and let

$$H(A, B) = \max\{\sup\{D(a, B) : a \in A\}, \sup\{D(b, A) : b \in B\}\},$$

be the Pompeiu–Hausdorff metric on $\mathcal{CB}(X)$ induced by the metric d, see [6]. Let $T : X \to \mathcal{P}(X)$ be a multivalued mapping. An element $x \in X$ such that $x \in T(x)$ is called a *fixed point* of T. We denote by $Fix(T)$ the set of all fixed points of T, i.e.,

$$Fix(T) = \{x \in X : x \in T(x)\}.$$

Let $f, g : X \to X$ be two self-mappings. A point $x \in X$ is a coincidence point of f, g and T iff $fx = gx \in Tx$. The set of coincidence points of f, g and T is denoted $\mathcal{COP}(f, g, T)$.

The following ladder of fixed point theorems for multi-valued mappings has been considered in [1].

Theorem 1 ([2]). *Let (X, d) be a complete metric space and $T : X \to \mathcal{CB}(X)$ a set-valued α-contraction, i.e., a mapping for which there exists a constant $\alpha \in (0, 1)$ such that*

$$H(Tx, Ty) \le \alpha\, d(x, y), \text{ for all } x, y \in X. \tag{2.1}$$

Then T has at least one fixed point.

Theorem 2 ([4]). *Let (X, d) be a complete metric space and $T : X \to \mathcal{CB}(X)$ a generalized multivalued (α, L)-weak contraction, i.e., a mapping for which there exists a function $\alpha : [0, \infty) \to [0, 1)$ satisfying $\limsup_{r\to t^+} \alpha(r) < 1$, for every $t \in [0, \infty)$, such that*

$$H(Tx, Ty) \le \alpha(d(x, y))d(x, y), \text{ for all } x, y \in X.$$

Then $Fix\,(T) \neq \emptyset$.

A function $\alpha : [0, \infty) \to [0, 1)$ satisfying $\limsup_{r\to t^+} \alpha(r) < 1$, for every $t \in [0, \infty)$, is usually called an $\mathcal{MT}$-function, see [1].

Theorem 3 ([3]). *Let (X, d) be a complete metric space and $T : X \to \mathcal{CB}(X)$ a generalized multivalued (α, L)-weak contraction, i.e., a mapping for which there exists $\mathcal{MT}$-function α and $L \ge 0$ such that*

$$H(Tx, Ty) \le \alpha(d(x, y))d(x, y) + LD(y, Tx), \text{ for all } x, y \in X. \tag{2.2}$$

Then $Fix\,(T) \neq \emptyset$.

Note that Theorem 3 is not correctly stated in [1]: condition (2.2) should be satisfied "for all $x, y \in X$" and not "for all $x, y \in X$ with $x \neq y$", as stated in [1].

The main result in [1] (Theorem 3.2) has been intended to generalize Theorem 1 [2], Theorem 2 [4], Theorem 3 [3] and some other related results, see [5] and [7]. It can be restated as follows.

Theorem 4 ([1]). *Let (X, d) be a complete metric space, $T : X \to \mathcal{CB}(X)$ be a multi-valued mapping, $f, g : X \to X$ be continuous self-mappings and $\alpha : [0, \infty) \to [0, 1)$ a $\mathcal{MT}$-function. Assume that*

(a_1) *for each $x \in X$, $\{fy = gy : y \in Tx\} \subset Tx$;*

(b_1) *there exist two mappings $\widehat{h}, \widehat{k} : X \to [0, +\infty)$ such that*

$$H(Tx, Ty) \le \alpha(d(x, y)) \cdot \frac{D(x, Ty) + D(y, Tx)}{2} + \widehat{h}(fy)D(fy, Tx) + \widehat{k}(gy)D(gy, Tx), \text{ for all } x, y \in X. \tag{2.3}$$

Then $\mathcal{COP}(f, g, T) \cap Fix\,(T) \neq \emptyset$.

In order to show the generality of Theorem 4 over Nadler's fixed point theorem (Theorem 1) and over Berinde–Berinde's fixed point theorem (Theorem 3), the authors considered in [1] the following example.

Example 1 (*[1], Example 3.3*)**.** Let $X = \{0, 3/4, 1\}$ and $d : X \times X \to \mathbb{R}$ be the usual metric. Let $T : X \to \mathcal{CB}(X)$ be a multi-valued mapping defined by $T(0) = \{0\}$, $T(3/4) = \{0, 3/4\}$, $T(1) = \{0, 1\}$ and $f, g : X \to X$ be given by $f = g = I_X$ (:= the identity mapping on X).

Define the function $\alpha : [0, \infty) \to [0, 1)$ by $\alpha(t) := 2/3$, for all $t \in [0, \infty)$ and let the two mappings $\widehat{h}, \widehat{k} : X \to [0, \infty)$ be given by $\widehat{h}(x) := 1$ and $\widehat{k}(x) := 0$, for all $x \in X$.

As shown in [1], T, and f, g as given in Example 1 satisfy all assumptions of Theorem 4. Since the inequality

$$H(T0, T1) = H(\{0\}, \{0, 1\}) = 1 > \alpha \cdot d(0, 1) = \alpha,$$

is true for any $\alpha \in (0, 1)$, we conclude that, indeed, Theorem 4 is more general than Nadler's fixed point theorem (Theorem 1).

But, in view of Example 3.3 [1], Theorem 4 is not more general than Berinde–Berinde's fixed point theorem (Theorem 3), because for $L = 1/3$ we have

$$H(T0, T1) = H(\{0\}, \{0, 1\}) = 1 \le \alpha \cdot d(0, 1) + LD(1, T0) = \frac{2}{3} + \frac{1}{3}.$$

(Note that the restriction $0 \le L < 1/3$ imposed by the authors in [1] is incorrect since, in the definition of a generalized multivalued (α, L)-weak contraction, L may be any positive constant, see [3] for more details and examples.) So, it remains an open problem whether or not Theorem 4 is more general than Theorem 3. For other very recent related results see also [8–10].

Acknowledgments

The first author's research was done during his visit to Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, in the period April–May 2016. He gratefully thanks Dr. Al-Homidan, dean of College of Sciences, and Dr. Al-Attas, the Chairman of the Department of Mathematics and Statistics, for the excellent conditions they offered during the visit. The second author would also like to acknowledge the support provided by the Deanship of Scientific Research at King Fahd University of Petroleum and Minerals for funding this work through the projects IN151014 and IN151017.

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