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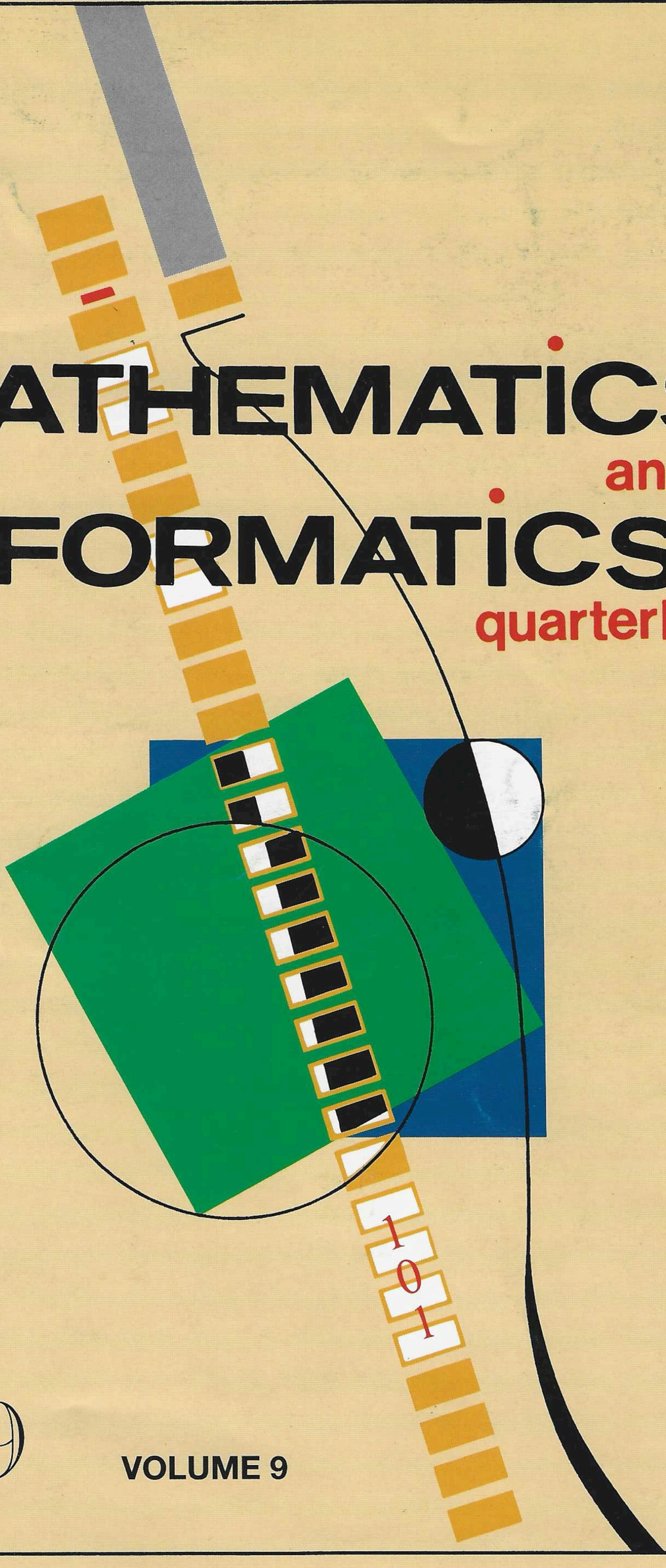

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PROBLEMS

SOLUTIONS

Let Us Discover Problems by Means of the Computer

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1. Introduction. An alternative title of this paper, suggested by the column "Pushing the limits" in *Mathematics and Informatics Quarterly*, could be: "Pushing the limits by means of the computer." Therefore, the main goal of this note is to show that the computer may be used not only for solving a (numerical) mathematical problem but also as an useful tool in exploring, discovering and posing new problems.

Problem 1. Find the digits a, b and c (in decimal system) so that

$$\overline{abc} \times \overline{abc} = \overline{nabc}, \tag{1}$$

where n is either a digit or a positive integer. (*Gazeta Matematica*, 3/1987, L. Parsan)

At the time I had solved the above problem, ten years ago, using pencil and paper, I was immediately tempted to try if (1) holds with other combinations of the three digits in the right side, e. g.

$$\overline{nbac},\ \overline{nbca},\ \overline{ncba},\ \overline{naaa},\ \overline{nbbb},\ \ldots \tag{2}$$

But how to explore, using pencil and paper, if (1) is still valid with a term from (2) in the right side? As at that time I was working as a computer programmer, the answer was very "professional": let's try with the computer.

The program, written in PASCAL, followed the usual steps in multiplying by hand the numbers $\overline{abc}$ and $\overline{abc}$, for all $a, b, c \in \{0, 1, 2, \ldots, 9\}$, $a \neq 0$, and checking the last three digits of the result. In this manner, it was obtained that for $\overline{nbac}$, $\overline{ncba}$ or $\overline{naaa}$ instead of $\overline{nabc}$ in the right side of (1), the equality holds and, moreover, in the last two cases the problem has a unique solution.

Some of the obtained problems were published in the Romanian journals *Gazeta Matematica* and *Revista Matematica din Timisoara.* Similar explorations were made around other problems. Here it is a sample of some of the discovered problems.

Problem 2. Find the digits a, b, c so that

$$\overline{abc} \times \overline{abc} = \overline{naaa},$$

where n is either a digit or a positive integer. (*Revista Mat. Tim.*, nr. 2/1987, V. Berinde)

Remark. The unique solution is $a = 4$, $b = 6$, $c = 2$ and $462 \times 462 = 213444$.

Problem 3. The same statement as in Problem 2 for

$$\overline{abc} \times \overline{abc} = \overline{nabc}.$$

(*Gazeta Matematica*, nr. 2/1988, V. Berinde)

Starting from another problem we obtained, after appropriate explorations made by means of a computer and by running a special program, the following problems.

Problem 4. Find the numbers $\overline{ba}$ in decimal representation so that $a + 8b$ divides $\overline{ab}$.
(*Gazeta Matematica*, 3/1988, V. Berinde)

Answer: $a = 7$, $b = 9$.

Problem 5. Find the numbers $\overline{ab}$ so that $9a + b$ divides $\overline{ba}$.

(*Revista Mat. Tim.*, nr. 2/1987, V. Berinde)

Answer: $a = 9$, $b = 8$.

Problem 6. Find the digits a, b, c, d in decimal system so that

$$\overline{cd} \times \overline{bcd} = \overline{abcd}.$$

(*Gazeta Matematica,* 9/1988, V. Berinde)

Answer: $a = 3$, $b = 1$, $c = 2$, $d = 5$.

Problem 7. Find the digits a, b, c so that $(a + b + c)^2$ divides both $\overline{abc}$ and $\overline{cba}$.
Answer: $a = 2$, $b = 4$, $c = 3$.

Problem 8. Find the digits a, b, c so that

$$\overline{ac} \times \overline{b1} = \overline{1abc}.$$

(*Gazeta Matematica,* 1/1992, V. Berinde)

Answer: $a = 8$, $b = 2$, $c = 7$.

Problem 9. Show that

$$123 \times 124 \times 125 \times 126 = \left(123 + \frac{3}{4}\right) \times \left(124 + \frac{8}{33}\right) \times \left(125 - \frac{3}{4}\right) \times \left(126 - \frac{18}{71}\right).$$

(*Gazeta Matematica,* 3/1989, V. Berinde)

Problem 10. Show that

$$117 \times 118 \times 119 \times 120 = \left(117 + \frac{3}{5}\right) \times \left(118 + \frac{11}{28}\right) \times \left(119 - \frac{3}{5}\right) \times \left(120 - \frac{15}{37}\right).$$

Remark. The explorations that lead to the last two problems were suggested by a problem in a Romanian textbook for the 5th grade.

The challenge for the readers of this note is not only to discover by themselves new problems by means of the computer but also to solve Problems 1–10 by classical means, that is, using pencil and paper.

Finally, we want to stress on the fact that all problems given above has been discovered by running an appropriate program written by ourselves on a computer and not by doing hundreds and hundreds of computations with a hand calculator.

For other problems obtained by computer see [1].

References

[1] Berinde, V., Let us propose mathematics problems by means of the computer, *Gazeta Matematica*, XCV (1990), Nr. 8–9, 232–236. (in Romanian)