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Why Pompeiu-Hausdorff metric instead of Hausdorff metric?

VASILE BERINDE and MĂDĂLINA PĂCURAR

ABSTRACT. The distance between two sets, commonly called *Hausdorff metric*, is a very important mathematical concept, with plenty of applications in almost all scientific research areas. We suggest in this paper an update of its name as *Pompeiu-Hausdorff metric (distance)*. Based on historical evidence, this proposal follows the contemporary manner of appointing concepts in scientific writings, especially in mathematics.

1. INTRODUCTION

If we are searching in MathScinet for ”Hausdorff metric” or ”Hausdorff distance” appearing in the title of the publications indexed there, we find more than 332 publications, while, if we are searching for the same terms appearing *Everywhere*, one gets a more impressive number: more than 4000 publications!

Now if we are searching in the same database but for ”Pompeiu-Hausdorff metric” or ”Pompeiu-Hausdorff distance” or ”Hausdorff-Pompeiu metric” or ”Hausdorff-Pompeiu distance” appearing in the title of the publications indexed there, we find the very modest number of 4 publications. Well, even if we are searching for these terms appearing *Everywhere*, one still gets a small number of publications: 83.

Let us now move to a general database, like Web of Science (Clarivate Analytics). When searching for ”Hausdorff metric” or ”Hausdorff distance” appearing in the title of the publications indexed, we find 478 results from Web of Science Core Collection, while, if we are searching for the same terms appearing in the *Topic*, one gets a really impressive number: more than 4500 publications!

Let’s do the same search but for ”Pompeiu-Hausdorff metric” or ”Pompeiu-Hausdorff distance” or ”Hausdorff-Pompeiu distance” or ”Hausdorff-Pompeiu metric” appearing in the title of the publications indexed in Web of Science. The result is really disappointing: 1 publication only. Despite this, let us search for the same terms appearing in the *Topic*: one finds 63 publications.

It is slightly better but... let’s do the same thing but this time for Scopus (Elsevier).

When searching for ”Hausdorff metric” or ”Hausdorff distance” appearing in the title of the publications indexed in SCOPUS, one finds 669 documents, while, when searching for the same terms but appearing in *Article title, Abstract, Key words* there are more than 5800 documents found.

Doing the same searches for the terms ”Pompeiu-Hausdorff metric” or ”Pompeiu-Hausdorff distance” or ”Hausdorff-Pompeiu distance” or ”Hausdorff-Pompeiu metric” produces 1 result, when searching in the title of the publications, and 66 results, when we are searching in *Article title, Abstract, Key words*.

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Corresponding author: Vasile Berinde; vasile.berinde@mi.utcluj.ro

But for what reason are we comparing in this way the "Hausdorff metric (distance)", on one side, to the "Pompeiu-Hausdorff metric (distance)" and its alternative "Hausdorff-Pompeiu metric (distance)", on the other side?

The answer is disarmingly simple: they all represent the same mathematical concept, i.e., the distance between two sets, a notion whose inception is due to Dimitrie Pompeiu (1873-1954), who introduced in 1905 the distance between two curves in $\mathbb{R}^2$, and which was formalized afterwards, in 1914, by Felix Hausdorff (1868-1942) to the general setting of a metric space.

So, starting from the disproportionality of the figures reported above, the aim of this note is to highlight Pompeiu's pioneering contribution in the process that eventually lead to the modern concept of distance between two sets in a metric space and thus to advocate for using the designation "Pompeiu-Hausdorff metric (distance)" instead of the dominant current name "Hausdorff metric (distance)".

2. Pompeiu's definition

D. Pompeiu defended his PhD thesis, entitled *On the continuity of functions of complex variables*, at the University of Paris (Sorbonne) in 1905, under the direction of Henry Poincaré. The thesis has been published in the same year, as was the custom at that time, in the journal *Annales de la Faculté de Sciences de Toulouse* (*Annals of Faculty of Sciences in Toulouse*) [26].

In his thesis, Pompeiu studied a problem formulated by Paul Painlevé [24] in 1897, concerning the singularities of uniform analytic functions. In this context, he defined the distance between two closed sets, but in the case of complex analysis.

In fact, Pompeiu has been interested to define the distance between two curves in the complex plane in order to rigorously define, on that base, the concept of limit of a sequence of sets (curves).

Before we present this concept let us first recall the definition of a *distance*.

Let X be a nonempty set (of points). A function $d : X \times X \to \mathbb{R}_+$ is called a *distance (metric)* if it satisfies the following three conditions:

(1) (positivity) $d(x, y) \geq 0$ and $d(x, y) = 0$ if and only if $x = y$;
(2) (symmetry) $d(x, y) = d(y, x)$, for all $x, y \in X$;
(3) (triangle inequality) $d(x, z) \leq d(x, y) + d(y, z)$, for all $x, y, z \in X$.

If d is a distance on X then we call the pair (X, d) a *metric space*. Note that here we have the *distance between two points*.

But what about the case when we are interested to define the *distance between two sets A* and B?

The first temptation would be to define it as the minimum value of all distances between the points $a \in A$ and the points $b \in B$, i.e., to consider the "distance" defined by

$$dist(A, B) = \inf\{d(a, b) : a \in A,\ b \in B\}, \tag{2.1}$$

see Figures 1 and 2.

But, it is obvious that such a "distance" does not satisfy the positivity property of a distance, see Figure 2, where $dist(A, B) = 0$ but obviously $A \neq B$!

Very probably, when Pompeiu was looking for an appropriate notion of distance between two sets, he would have considered the previous idea.

Anyway, no matter how he searched, in the end he proceeded as follows, see his *Opere Complete (Complete Works)* [27], page 12, where the whole content of [26] has been reprinted.

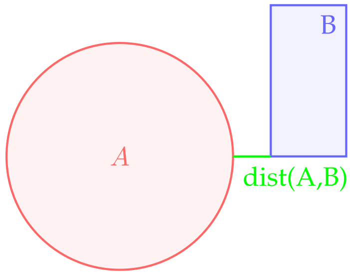


FIGURE 1. The "distance" between A and B, when $A \cap B = \emptyset$

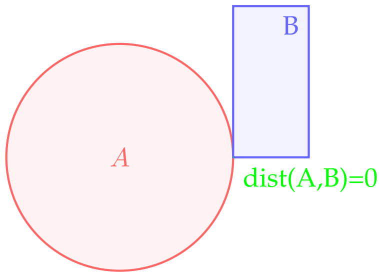
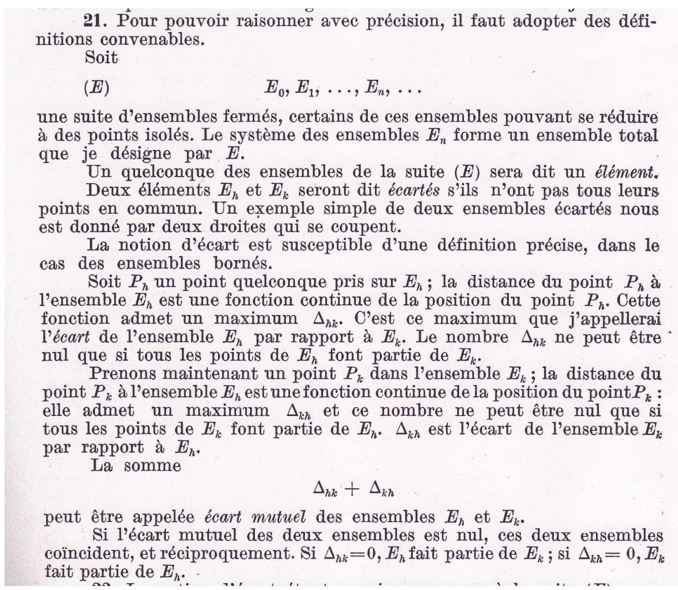


FIGURE 2. The "distance" between A and B, when $A \cap B \neq \emptyset$



21. Pour pouvoir raisonner avec précision, il faut adopter des définitions convenables.

Soit

$$(E) \qquad E_0, E_1, \ldots, E_n, \ldots$$

une suite d'ensembles fermés, certains de ces ensembles pouvant se réduire à des points isolés. Le système des ensembles E_n forme un ensemble total que je désigne par E.

Un quelconque des ensembles de la suite (E) sera dit un *élément*.

Deux éléments E_h et E_k seront dit *écartés* s'ils n'ont pas tous leurs points en commun. Un exemple simple de deux ensembles écartés nous est donné par deux droites qui se coupent.

La notion d'écart est susceptible d'une définition précise, dans le cas des ensembles bornés.

Soit P_h un point quelconque pris sur E_h ; la distance du point P_h à l'ensemble E_k est une fonction continue de la position du point P_h. Cette fonction admet un maximum Δ_{hk}. C'est ce maximum que j'appellerai l'*écart* de l'ensemble E_h par rapport à E_k. Le nombre Δ_{hk} ne peut être nul que si tous les points de E_h font partie de E_k.

Prenons maintenant un point P_k dans l'ensemble E_k ; la distance du point P_k à l'ensemble E_h est une fonction continue de la position du point P_k : elle admet un maximum Δ_{kh} et ce nombre ne peut être nul que si tous les points de E_k font partie de E_h. Δ_{kh} est l'écart de l'ensemble E_k par rapport à E_h.

La somme

$$\Delta_{hk} + \Delta_{kh}$$

peut être appelée *écart mutuel* des ensembles E_h et E_k.

Si l'écart mutuel des deux ensembles est nul, ces deux ensembles coïncident, et réciproquement. Si $\Delta_{hk}=0$, E_h fait partie de E_k ; si $\Delta_{kh}=0$, E_k fait partie de E_h.

FIGURE 3. Copy of of Pompeiu's thesis where $P(A;B)$ is introduced

Naturally, we are using the current terminology and notations to transcribe his approach.

Let A, B be two closed and bounded sets. If $a \in A$, then the distance between the point a and the set B is by definition

$$d(a,B) = \min\{d(a,b) : b \in B\},$$

where $d(a, b)$ is the (Euclidean) distance between the points a and b (do not forget that Pompeiu was working in the complex plane!).

Further, Pompeiu defined the *asymmetric distance* (écart, in French) between the sets A and B as

$$D(A, B) = \max\{d(a, B) : a \in A\}.$$

He noted that $D(A, B) = 0$ if and only if "all points of A belong to B", that is, $A \subset B$.

Therefore, he also considered the *asymmetric distance* (écart) between the sets B and A, as

$$D(B, A) = \max\{d(b, A) : b \in B\},$$

and noted that $D(B, A) = 0$ if and only if $B \subset A$.

We can easily see from Figures 3 and 4 that, in general,

$$D(A, B) \text{ (length of green segment)} \neq D(B, A) \text{ (length of black segment)}.$$

This means that D does not satisfy the symmetry property, which is essential to qualify D for being a true distance!

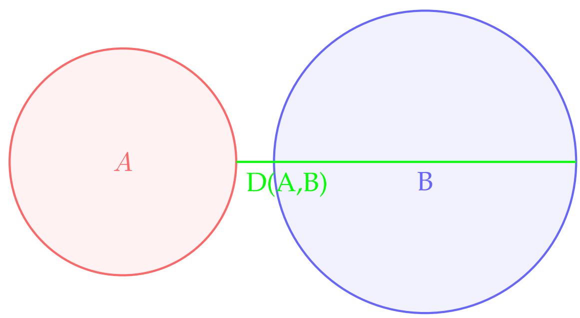


FIGURE 4. The asymmetric distance between A and B: $D(A, B)$

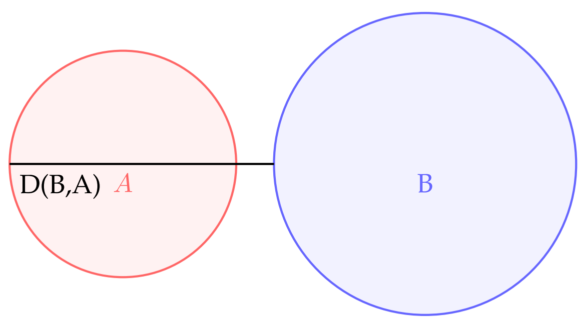


FIGURE 5. The asymmetric distance between B and A: $D(B, A)$

Hence, in order to endow the distance between two sets with this natural property, i.e., with the *symmetry*, Pompeiu considered a very natural way to symmetrize his concept, by defining the *distance between the sets A and B* (écart mutuel, in French), denoted here by $P(A, B)$, by

$$P(A, B) = D(A, B) + D(B, A). \tag{2.2}$$

He also concluded that $P(A, B) = 0$ if and only if $D(A, B) = 0$ and $D(B, A) = 0$, that is, if and only if $A = B$.

Therefore, so far it is clear that Pompeiu's distance P satisfies the first two properties of a true distance: positivity and symmetry. As for the third property, he did not need it in his study so this one was not discussed.

Let us end this section by illustrating how one computes the Pompeiu distance between some particular sets in $\mathcal{P}(\mathbb{R})$ and $\mathcal{P}(\mathbb{R}^2)$.

Example 2.1. We consider on the real line the sets $A = [0, 1]$; $B = [2, 5]$ (see Figure 6). Recall that the distance between a point x and a set A is defined as

$$d(x, A) = \inf\{d(x, a) : a \in A\} \; (= \min\{d(x, a) : a \in A\}).$$

We proceed as follows:
1. We compute $\{d(a, B) : a \in A\} = [1, 2]$ and deduce that $\max\{d(a, B) : a \in A\} = 2$. This shows that

$$D(A, B) = 2.$$

2. We compute $\{d(b, A) : b \in B\} = [1, 4]$ and deduce that $\max\{d(b, A) : b \in B\} = 4$. Hence

$$D(B, A) = 4.$$

3. Now, Pompeiu's distance between A and B is obtained by (2.2), i.e.,

$$P(A, B) = D(A, B) + D(B, A) = 2 + 4 = 6.$$

FIGURE 6. The sets $A = [0, 1]$ and $B = [2, 5]$ in Example 2.1

Example 2.2. Let $A = [0, 2]$ and $B = [1, 5]$. In a similar manner to Example 2.1 we have:
1. $\{d(a, B) : a \in A\} = [0, 1]$, hence $\max\{d(a, B) : a \in A\} = 1$ and

$$D(A, B) = 1.$$

2. $\{d(b, A) : b \in B\} = [0, 3]$ and $\max\{d(b, A) : b \in B\} = 3$. So

$$D(B, A) = 3.$$

3. Therefore, Pompeiu's distance between A and B is:

$$P(A, B) = D(A, B) + D(B, A) = 1 + 3 = 4.$$

Note that, in this case, $A \cap B = [1, 2]$ and if we would apply formula (2.1) we would obtain $dist(A, B) = 0$, despite the fact that $A \neq B$!

FIGURE 7. The sets $A = [0, 2]$ and $B = [1, 5]$ in Example 2.2

Example 2.3. We consider now the sets A and B to be the squares given by

$$A = \{(x, y) : x \in [-1, 0],\, y \in [-1, 0]\}; \quad B = \{(x, y) : x \in [0, 2],\, y \in [0, 2]\}$$

in $\mathcal{P}(\mathbb{R}^2)$.

Keeping in mind that the diagonal of A has the length $\sqrt{2}$, while the diagonal of B has the length $2\sqrt{2}$, we have

1. $\{d(a,B) : a \in A\} = [0, \sqrt{2}]$, so $\max\{d(a,B) : a \in A\} = \sqrt{2}$ and
$$D(A,B) = \sqrt{2}.$$
2. $\{d(b,A) : b \in B\} = [0, 2\sqrt{2}]$, hence $\max\{d(b,A) : b \in B\} = 2\sqrt{2}$ and
$$D(B,A) = 2\sqrt{2}.$$
3. Therefore, the Pompeiu's distance between A and B is
$$P(A,B) = D(A,B) + D(B,A) = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}.$$

Note that, in this case, $A \cap B = \{(0,0)\}$ and if we would apply formula (2.1) we would obtain $dist(A,B) = 0$, despite the fact that $A \neq B$!

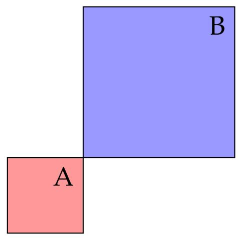


FIGURE 8. The sets A and B in Example 2.3

3. HAUSDORFF'S SYMMETRIZATION

In the previous section we have briefly presented Pompeiu's approach in defining a distance between two sets. In the current section we aim to see what part of this approach has been further extended by Hausdorff in his famous book [11] and how exactly.

First of all, Hausdorff took the chance of benefiting from a fundamental mathematical contribution from the beginning of the 20th century: the concept of a metric space, introduced by the French mathematician Maurice Fréchet (1878-1973) in his PhD thesis [9] defended in 1906.

Therefore, Hausdorff worked in the general setting of a metric space, where he considered all the basic concepts introduced by Pompeiu with respect to the distance between two sets, and he also proved rigorously the three fundamental properties of the distance function, including the triangle inequality.

Secondly, Hausdorff adopted an alternative way to symmetrize the asymmetric distances $D(A,B)$ and $D(B,A)$, by defining what is currently denoted by $H(A,B)$ and commonly named *Hausdorff metric*:
$$H(A,B) = \max\{D(A,B), D(B,A)\}. \tag{3.3}$$
The two definitions due to Pompeiu and Hausdorff are equivalent, by virtue of the double inequality
$$\frac{1}{2} \cdot (u+v) \leq max\{u,v\} \leq 1 \cdot (u+v),$$
which yields
$$\frac{1}{2} \cdot P(A,B) \leq H(A,B) \leq 1 \cdot P(A,B).$$
If we use Hausdorff's symmetrization formula (3.3), then for the sets in Examples 2.1-2.3 the distances will be the following ones:

1. For A and B in Example 2.1, $H(A,B) = 4$;

2. For A and B in Example 2.2, $H(A,B) = 3$;

3. For A and B in Example 2.3, $H(A,B) = 2\sqrt{2}$.

It is important to say that Hausdorff cited honestly Pompeiu's contribution: in the first edition of his book [11] (at page 463), in its shorter second edition [12] (at page 280), as well as in the third edition [13] and its two translations (Russian translation [14], at page 293 and English translation [17], at page 343) and in this way he explicitly acknowledged Pompeiu's priority.

What is then the reason why, even if Hausdorff explicitly mentioned Pompeiu's priority, a fact confirmed in the monograph of Kuratowski from 1933 [21], and Pompeiu's work was mentioned in the same way in [11], [12] and [13] (and its translations [14] and [17]), the posterity however credited only Hausdorff as creator of this fundamental concept?

In our opinion, the first explanation is that Hausdorff's book *Set Theory* was a fundamental reference for more than 50 years by its novelty, rigour and readability, and so it has been very influential for many generations of mathematicians, mainly due to its Russian, Polish and English translations.

Secondly, Hausdorff referred to the contributions of Pompeiu in a manner that was not so visible, but was more common for the scientific publications of that period.

We illustrate this fact by taking as an example the first edition of Hausdorff's book [11].

In this book and then in all its subsequent editions and translations, there is no any explicit citation in the text to acknowledge Pompeiu's priority.

Instead, at the end of the book there is an Annex or Addenda (*Anhang*, in German) with Supplements and Notes (*Nachträge und Anmerkung*), where for each section and paragraph there are written some notes and comments with respect to various aspects, including citations of prior contributions, by simply mentioning the paragraph and page to which the comment/note refers to.

So, at page 463 of the book [11], for §6, page 293 (where it was defined the distance between two sets A and B, denoted by $\overline{AB}$), Hausdorff has written the following note:

Zur Definition von $\overline{AB}$ vgl. D. Pompéju, Sur la continuité de fonctions de variables complexes, Ann. Fac. Toulouse (2) 7 (1905).

which means

For the definition of $\overline{AB}$ see D. Pompéju, Sur la continuité de fonctions de variables complexes, Ann. Fac. Toulouse (2) 7 (1905).

That's all.

Now, let us try to guess the answer to the following simple question: How many readers of Hausdorff's book might have read the above note?

Not many, for sure...

So, it is not surprising that Hausdorff has been credited as the unique author of the concept of distance between two sets and that the symmetrization formula he proposed has been adopted and used by almost all authors.

4. Conclusion

Dimitrie Pompeiu's contribution in defining the distance between two sets has been outlined in many books, such as:

- Istrăţescu, V. I. *Fixed point theory. An introduction*. With a preface by Michiel Hazewinkel. Mathematics and its Applications, 7. D. Reidel Publishing Co., Dordrecht-Boston, Mass., 1981.
- Bânzaru, T.; Rendi, B. *Topologies on spaces of subsets and multivalued mappings*. Mathematical Monographs/Monografii Matematice, 63. University of Timişoara, Department of Mathematics, Timişoara, 1997.
- Rockafellar, R. T.; Wets, R. J.-B. *Variational analysis*. Grundlehren der mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 317. Springer-Verlag, Berlin, 1998.
- Aubin, J.-P. *Mutational and morphological analysis. Tools for shape evolution and morphogenesis*. Systems & Control: Foundations & Applications. Birkhäuser Boston, Inc., Boston, MA, 1999.
- Petruşel, A.; Moţ, G. *Multivalued analysis and mathematical economics*. House of the Book of Science, Cluj-Napoca, 2004.
- Dontchev, A. L.; Rockafellar, R. T. *Implicit functions and solution mappings. A view from variational analysis*. Springer Monographs in Mathematics. Springer, Dordrecht, 2009.
- Lorenz, T. *Mutational analysis. A joint framework for Cauchy problems in and beyond vector spaces*. Lecture Notes in Mathematics, 1996. Springer-Verlag, Berlin, 2010.
- Dontchev, A. L.; Rockafellar, R. T. *Implicit functions and solution mappings. A view from variational analysis*. Second edition. Springer Series in Operations Research and Financial Engineering. Springer, New York, 2014.

There appeared also papers in which some efforts were made to draw attention on Pompeiu's contribution, see for example [10], [22], [35], [5], [36], [4] and the more recent paper [2] (that has actually motivated us to write the present paper, in order to fulfill an obligation).

So this idea of updating the name of *Hausdorff metric (distance)* to that of *Pompeiu-Hausdorff metric (distance)* is not a new one and it doesn't express a personal taste. It is based on historical evidence and it follows the manner in which mathematical contributions are acknowledged in nowadays scientific writings.

By the present paper we hope to get the attention of researchers using the distance between two sets and to spread the proposed update in their future works.

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
NORTH UNIVERSITY CENTRE AT BAIA MARE
TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
VICTORIEI 76, 430072 BAIA MARE, ROMANIA
Email address: vasile.berinde@mi.utcluj.ro

DEPARTMENT OF ECONOMICS AND BUSSINESS ADMINISTRATION IN GERMAN LANGUAGE
FACULTY OF ECONOMICS AND BUSSINESS ADMINISTRATION
BABEŞ-BOLYAI UNIVERSITY OF CLUJ-NAPOCA
58-60 T. MIHALI ST., 400591 CLUJ-NAPOCA, ROMANIA
Email address: madalina.pacurar@econ.ubbcluj.ro